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# CONTROL OF STRUCTURES WITH CUBIC AND QUADRATIC NON-LINEARITIES WITH TIME DELAY CONSIDERATION

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***Abstract.*** *This paper studies the effect of time delay on the active non-linear control of dynamically loaded flexible structures. The behavior of non-linear systems under state feedback control, considering a fixed time delay for the control force, is investigated. A control method based on non-linear optimal control, using a tensorial formulation and state feedback control is used. The state equations and the control forces are expressed in polynomial form and a performance index, quadratic in both state vector and control forces, is used. General polynomial representations of the non-linear control law are obtained and implemented for control algorithms up to the fifth order. This methodology is applied to systems with quadratic and cubic non-linearities. Strongly non-linear systems are tested and the effectiveness of the control system, including a delay in the application of control forces is discussed. Numerical results indicate that the control adopted algorithm can be efficient for non-linear systems, chiefly in the presence of strong non-linearities. On the other hand increasing time delay reduces the efficiency of the control system. This emphasizes the importance of considering time delay in the design of active structural control systems.*

***Key words:*** *Time delay, non-linear optimal control, active control systems.*

## 1. INTRODUCTION

In recent years, considerable attention has been paid to active structural control research. Particularly over the last two decades, remarkable progress has been made in research on using active and hybrid control systems as a means of structural protection against wind, earthquakes and other hazards (Soong, 1990, Spencer Jr 1996). Most of these studies employ linear control strategies. Non-linear structural control has a more recent history, but some interesting strategies have been proposed lately (Yang et al. 1988 and 1994). The investigation of time delayed actively controlled structural systems is also relatively recent but the importance of considering its effects on the design of control systems has been showed by some researchers (Roorda, 1980, Abdel-Rohman, 1987, Agrawal and Yang, 1997).

In a paper presented in the last COBEM, the authors of this paper used a methodology for the non-linear active control of flexible structures, in order to limit the amplitudes of oscillations within safe allowable bounds (Pinto and Gonçalves, 1997). This control method was based on non-linear optimal control theory, used an indicial formulation and state feedback control (Tomasula et al., 1996, Pinto and Gonçalves, 1999) and assumed an idealized system without any time delay. In real systems there are unavoidable time delays. Actually, time delay is one of the main issues concerning the use of real active structural control systems. It is due to the time necessary for data acquisition and conditioning, computing the required control forces, generating and transmitting the signal to the actuators, and applying the control forces to the structure (reaction time of actuators). The magnitude of the time delay is expected to decrease as more advanced control system software and hardware become available, but, even with the advances of technology, time delay can not be eliminated; only minimized. Therefore, it is an intrinsic parameter and should be considered in the project of active control systems. Time delay induces a phase shift which may degrade the performance of the control system and, if it is not handled properly, can not only render it ineffective but also cause instability of the controlled system. In this paper, the effects of time delay on the controlled system is studied. Numerical studies for a single degree of freedom system with quadratic and cubic non-linearities under state feedback control and different kinds of loading are presented and analyzed. Results emphasize the importance of considering time delay in the design of active structural control systems.

## 2. PROBLEM FORMULATION

### 2.1 Control strategy

Using an indicial formulation of tensor algebra (Suhardjo et al. 1993, Pinto & Gonçalves, 1998), the state equations of a certain class of nonlinear autonomous controlled systems can be expressed in the polynomial form

$$\dot{x}^{\mathrm{i}} = A^{\mathrm{i}}_{\mathrm{j}} x^{\mathrm{j}} + A^{\mathrm{i}}_{\mathrm{jk}} x^{\mathrm{jk}} + A^{\mathrm{i}}_{\mathrm{jkl}} x^{\mathrm{jkl}} + \ldots + B^{\mathrm{i}}_{\mathrm{j}} u^{\mathrm{j}} \tag{1}$$

where $x^{\mathrm{i}}$ is the i-th state variable, $A^{\mathrm{i}}_{\mathrm{j}}$, $A^{\mathrm{i}}_{\mathrm{jk}}$, $A^{\mathrm{i}}_{\mathrm{jkl}}$, ... are coefficients related to system's properties, $u^{\mathrm{j}}$ is the j-th control force and $B^{\mathrm{i}}_{\mathrm{j}}$ is a coefficient that relates $u^{\mathrm{j}}$ with $x^{\mathrm{i}}$. Here we define $x^{\mathrm{ij}} = x^{\mathrm{i}}x^{\mathrm{j}}$, $x^{\mathrm{ijk}} = x^{\mathrm{i}}x^{\mathrm{j}}x^{\mathrm{k}}$, and so on.

Assuming state feedback, the control forces can be expressed as

$$u^{\mathrm{i}} = K^{\mathrm{i}}_{\mathrm{j}} x^{\mathrm{j}} + K^{\mathrm{i}}_{\mathrm{jk}} x^{\mathrm{jk}} + K^{\mathrm{i}}_{\mathrm{jkl}} x^{\mathrm{jkl}} + \ldots \tag{2}$$

where $K^{\mathrm{i}}_{\mathrm{j}}$, $K^{\mathrm{i}}_{\mathrm{kj}}$, $K^{\mathrm{i}}_{\mathrm{jkl}}$, ... are the i-th gains of control order 1, 2, 3, ..., respectively.

The adopted performance index has the general form

$$J = \int_{t_0}^{t_f} (R_{\mathrm{ij}} u^{\mathrm{ij}} + Q_{\mathrm{ij}} x^{\mathrm{ij}} + Q_{\mathrm{ijk}} x^{\mathrm{ijk}} + Q_{\mathrm{ijkl}} x^{\mathrm{ijkl}} + \ldots)\, dt \tag{3}$$

where $t_0$ is the initial time, usually set to zero, $t_{\mathrm{f}}$ is the final time, $Q_{\mathrm{ij}}$, $Q_{\mathrm{ijk}}$, $Q_{\mathrm{ijkl}}$, ... are symmetric positive semi-definite tensors with order 2, 3, 4, ..., respectively, and $R_{\mathrm{ij}}$ is a symmetric positive definite tensor of second order. **Q** and **R** are weighting tensors, with elements chosen depending on the relative importance attributed to state variable bounds and control forces. High values of $Q_{\mathrm{ij}}$, $Q_{\mathrm{ijk}}$, ..., stress the reduction of system response, while high values of $R_{ij}$ result in less control effort (less energy consume). Choosing these values appropriately one can get a control as efficient as possible, without a great energy consumption.

Expressing the performance index by Taylor series

$$J = V_{ij} x^{ij} + V_{ijk} x^{ijk} + V_{ijkl} x^{ijkl} + ... \tag{4}$$

where $V_{ij}$, $V_{ijk}$, $V_{ijkl}$, etc... are symmetric related to their indices, one can obtain by a minimisation procedure the following Hamilton-Jacobi-Bellman equation

$$\min_{u}\left[ g(\mathbf{x},\mathbf{u},t) + \left(\frac{\partial J}{\partial \mathbf{x}}\right)^{T} \mathbf{f}(\mathbf{x},\mathbf{u},t) + \frac{\partial J}{\partial t}\right] = 0 \tag{5}$$

where $J$ is given by Eq. 4 and

$$\mathbf{f}(\mathbf{x},\mathbf{u},t) = \dot{\mathbf{x}}(t) \tag{6.a}$$

$$g(\mathbf{x},\mathbf{u},t) = R_{ij} u^{ij} + Q_{ij} x^{ij} + Q_{ijk} x^{ijk} + Q_{ijkl} x^{ijkl} + ... \tag{6.b}$$

Manipulating Eqs. 1, 4, 5 and 6, the first order control gain and the corresponding control equation, known as the Riccati equation, can be obtained:

$$K_i^a = -\left(R^{-1}\right)_{am} V_{bi} B_m^b \tag{7}$$

$$\mathrm{sym}\left[\dot{V}_{ij} + 2V_{ai} A_j^a - V_{ai} B_b^a \left(R^{-1}\right)_{cb} V_{dj} B_c^d + Q_{ij}\right] = 0 \tag{8}$$

where sym is the symmetry operator, defined in a way that when acting on a tensor **T** represents its symmetric form with respect to the free indices, i.e.

$$sym\left[T_{ij}\right] = \frac{1}{2!}\left(T_{ij} + T_{ji}\right),\ sym\left[T_{ijk}\right] = \frac{1}{3!}\left(T_{ijk} + T_{kij} + T_{jki} + T_{ikj} + T_{jik} + T_{kji}\right),\ ... \tag{9}$$

Once Vij is obtained from Eq. 8, one can get the first order control gains from Eq. 7. So the control forces can be computed with Eq. 2, resulting in the classical linear optimal control formulation. Similarly, one can get the equations for the second, third, fourth and fifth orders control, using the following equations:

a)second order:

$$\mathrm{sym}\left[\dot{V}_{ijk} + 2V_{ai} A_{jk}^a + 3V_{aij}\left(A_k^a + B_b^a K_k^b\right) + Q_{ijk}\right] = 0 \tag{10}$$

$$K_{ij}^a = -\frac{3}{2}\left(R^{-1}\right)_{ab} V_{cij} B_b^c \tag{11}$$

b)third order:

$$\begin{aligned} &\mathrm{sym}[\dot{V}_{ijkl} + 2V_{ai} A_{jkl}^a + 3V_{aij}\left(A_{kl}^a + B_b^a K_{kl}^b\right) + 4V_{aijk}\left(A_l^a + B_b^a K_l^b\right) + \\ &+ K_{ij}^a R_{ab} K_{kl}^b + Q_{ijkl}] = 0 \end{aligned} \tag{12}$$

$$K^{a}_{ijk} = -2\left(R^{-1}\right)_{ab} V_{cijk} B^{c}_{b} \tag{13}$$

c)fourth order:

$$\begin{aligned}&\mathrm{sym}[\dot{V}_{ijklm} + 2V_{ai} A^{a}_{jklm} + 3V_{aij}\left(A^{a}_{klm} + B^{a}_{b} K^{b}_{klm}\right) + 4V_{aijk}\left(A^{a}_{lm} + B^{a}_{b} K^{b}_{lm}\right) + \\ &+5V_{aijkl}\left(A^{a}_{m} + B^{a}_{b} K^{b}_{m}\right) + 2K^{a}_{ij} R_{ab} K^{b}_{klm} + Q_{ijklm}] = 0\end{aligned} \tag{14}$$

$$K^{a}_{ijkl} = -\frac{5}{2}\left(R^{-1}\right)_{ab} V_{cijkl} B^{c}_{b} \tag{15}$$

d)fifth order:

$$\begin{aligned}&\mathrm{sym}[\dot{V}_{ijklmn} + 2V_{ai} A^{a}_{jklmn} + 3V_{aij}\left(A^{a}_{klmn} + B^{a}_{b} K^{b}_{klmn}\right) + 4V_{aijk}\left(A^{a}_{lmn} + B^{a}_{b} K^{b}_{lmn}\right) + \\ &+5V_{aijkl}\left(A^{a}_{mn} + B^{a}_{b} K^{b}_{mn}\right) + 6V_{aijklm}\left(A^{a}_{n} + B^{a}_{b} K^{b}_{n}\right) + 2K^{a}_{ij} R_{ab} K^{b}_{klmn} + \\ &+K^{a}_{ijk} R_{ab} K^{b}_{lmn} + Q_{ijklmn}] = 0\end{aligned} \tag{16}$$

$$K^{a}_{ijkl} = -3\left(R^{-1}\right)_{ab} V_{cijklm} B^{c}_{b} \tag{17}$$

and so on, up to the desired order. These equations have a well defined pattern, in a way that one can get higher order control equations without difficulty.

In order to get the tensors **V,** one has to solve numerically a set of differential equations, with a certain computational cost. However, in most structural problems, $t_f$ is much longer then the natural period and setting it to ∞ doesn't change considerably the results and simplify significantly the problem, converting the differential equations into algebraic ones. In this work we derived equations for control up to the fifth order using this methodology.

## 2.2 Single degree of freedom non-linear systems

In this section, the application of the strategy presented in the previous section for single degree of freedom (sdof) systems with quadratic and cubic nonlinearities is studied. These models are capable of representing approximately, at least in a qualitative way, most of the elements usually used in civil and mechanical engineering structures, such as beams, plates, shells and arches. The equation of motion of such sdof nonlinear controlled autonomous systems can be expressed as

$$\ddot{x} + 2\mu\dot{x} + \omega^2 x + \alpha x^2 + \beta x^3 + \gamma u = 0 \tag{18}$$

where $x$ is the displacement, $\dot{x}$, the velocity, $\ddot{x}$, the acceleration, μ , the damping coefficient, ω, the natural frequency of the system, α and β, the quadratic and cubic nonlinear coefficients, respectively, and γ, the coefficient of the control force, $u$.

Using the formulation presented in the previous section, the state equations can be expressed as in Eq. 1 with the state variables $x^1 = x$ and $x^2 = \dot{x}$, and the control force $u^1 = u$, here a scalar. In this case, the only non-zero coefficients are $A^2_1 = -\omega^2$, $A^1_2 = 1$, $A^2_2 = -2\mu$, $A^2_{11} = -\alpha$, $A^2_{111} = -\beta$ and $B^2_1 = -\gamma$.

Assuming the performance index

$$J = \frac{1}{2}\int_0^\infty \left[\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u}\right]dt \tag{19}$$

the first order control gains for displacement and velocity are given by

$$K_1^1 = \frac{1}{\gamma}\left(-\omega^2 + \sqrt{\omega^4 + \gamma^2 \frac{Q_{11}}{R_{11}}}\right) \tag{20}$$

$$K_2^1 = \frac{1}{\gamma}(-2\mu + \sqrt{4\mu^2 + \gamma^2 \frac{Q_{22}}{R_{11}} + 2\left(-\omega^2 + \sqrt{\omega^4 + \gamma^2 \frac{Q_{11}}{R_{11}}}\right)}) \tag{21}$$

which are the same obtained using the algebraic Riccati equations.

Using these results, the second order control gain for displacement takes the form

$$K_{11}^1 = \frac{-\alpha}{\gamma}\left(1 - \frac{\omega^2}{\sqrt{\omega^4 + \gamma^2 \frac{Q_{11}}{R_{11}}}}\right) \tag{22}$$

Control gains of higher order can be obtained in a similar way. Here we developed a computer code capable of generating the gains up to the fifth order.

It is interesting to note that when α is null we have the well known Duffing equation, object of study of a number of nonlinear control works (Hackl et al. 1993, Cheng et al. 1993, Cui et al. 1997, Yabuno, 1997). In such case, the second order ($K_{\mathrm{jk}}^{\mathrm{i}}$) and forth order ($K_{\mathrm{jklm}}^{\mathrm{i}}$) gains are all null, and the third order gain for displacement is given by

$$K_{111}^1 = \frac{-\beta}{\gamma}\left(1 - \frac{\omega^2}{\sqrt{\omega^4 + \gamma^2 \frac{Q_{11}}{R_{11}}}}\right) \tag{23}$$

## 3. NUMERICAL EXAMPLE

In this section the control strategy presented in the previous sections is applied to the problem of a very shallow pressure loaded spherical cap, first for the system without any delay and afterwards considering input time delay. Unit values for γ and $R_{11}$ were adopted, and in **Q** only $Q_{11}$ is not null.

For such a problem, the first mode is dominant and a simplified one-degree-of-freedom model is capable of describing with a reasonable degree of accuracy the nonlinear behavior of the cap (Gonçalves, 1994). The same pressure loaded thin-walled spherical shell presented in Gonçalves (1994) is considered. The dimensionless sdof equation of motion modeling the vertical displacement $w$ is given by

$$\ddot{w} + \dot{w} + 219{,}9w - 410{,}5w^2 + 154{,}2w^3 = F(t) \qquad (24)$$

This system has a two-well potential function, with two stable equilibrium states at $w$ = 0 (the reference state) and at $w$ = 1,92. The final state of the systems depends on the initial conditions and load characteristics, as can be observed in Fig. 1, where the free vibration response of the cap is shown for two different sets of initial conditions, $w(0)$ = 0,70 and $w(0)$ = 0,75, both with $\dot{w}(0) = 0$.

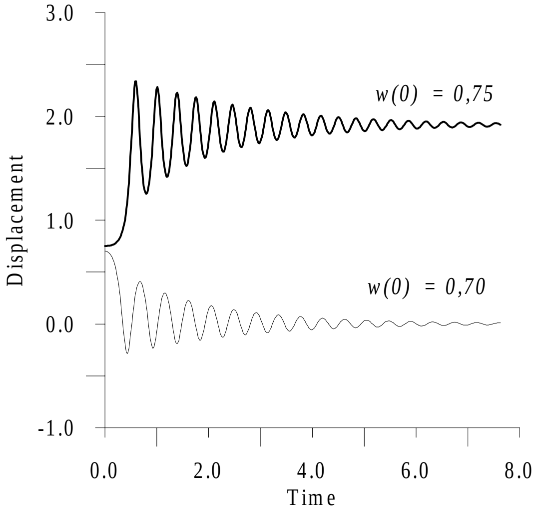


Figure 1: Free vibration response.

Depending on the force characteristics, the response will be attracted to one of the two potential wells, corresponding to pre and post-buckling equilibrium positions. For example, for an harmonic excitation $F(t)$ = $F_0$ sin(12$t$), the uncontrolled system escape at a load magnitude $F_o = 10{,}76$, as shown in Fig. 2.a. For an impact load, $F(t)$ = $F_0$, the cap jumps to the second potential well when $F_0$ reaches the escape load, here 26,87, as observed in Fig. 2.b.

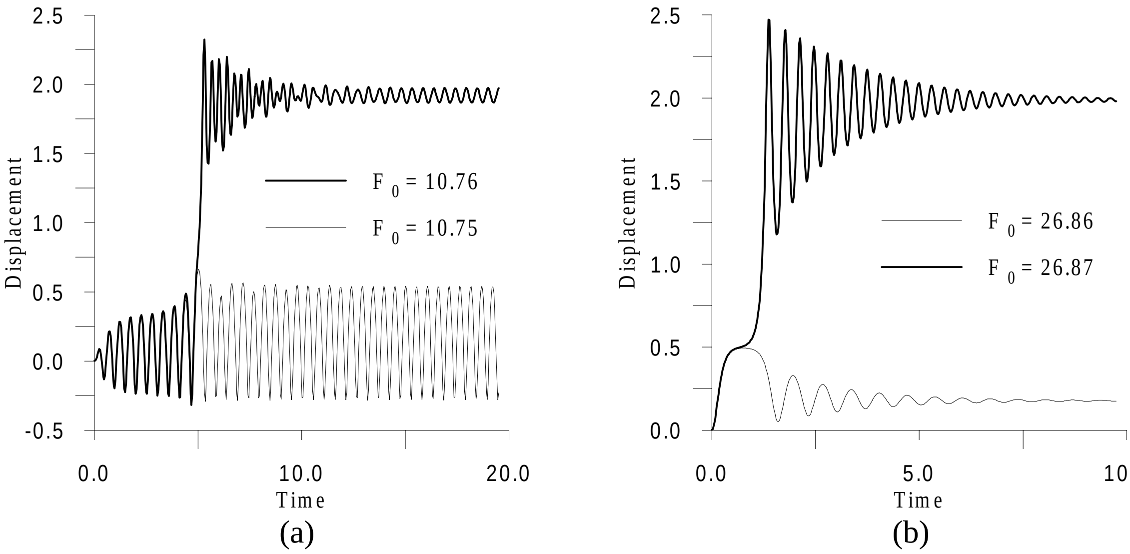


Figure 2. System without control. a) harmonic load, b) impact (step) load.

### 3.1 Controlled system without time delay

An adequate control system can be used to prevent escape (dynamic buckling). Figure 3 shows the response of the structure without any control and with controls of order two and three ($Q_{11}$ = 500), respectively, for a load amplitude of 15, about 40% higher than the escape load without control. One can see that only a third order control (or higher) can avoid the failure of the structure. Higher order algorithms can be more efficient, resulting in smaller displacements without increasing the energy consumption. The peak control force for the third order algorithm is 7,41, very small if compared to the static critical load (204,9).

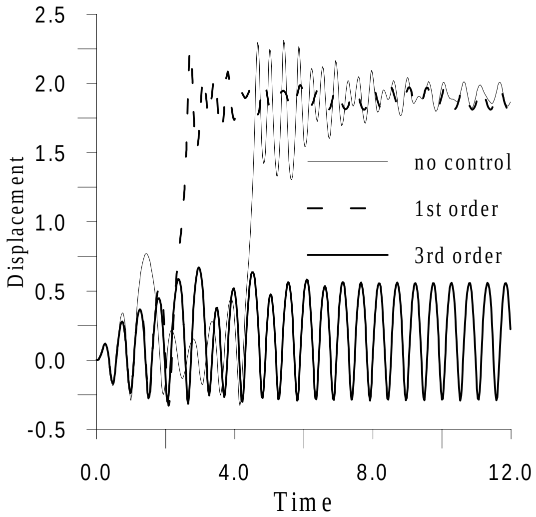


Figure 3. Time response for the load $F(t) = 15 \sin(12t)$.

Table 1 shows how the escape load increases with the order of the adopted control system when compared with the escape load of the system without any control ($F_0$ = 10,76).

Table 1. Variation of the escape load with the order of the control system (harmonic load).

| Order | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $F_0$ | 13,49 | 14,55 | 16,96 | 16,99 | 20,99 |
| $\Delta F_0$ (%) | 25,37 | 35,22 | 57,62 | 57,99 | 95,07 |

One can see that a higher order control system results in higher escape loads, enlarging the safe working capacity of the structure without demanding great control forces.

The same system was also subjected to impact (step) loading, $F(t) = F_0$, resulting in a escape load of 26,87. Figures 4.a and 4.b show, respectively, the time response and control forces demanded for a step load of magnitude 35, more then 30% higher than the escape load for the structure without control. Here $Q_{11}$ = 3000.

For this load level, a control algorithm of order less than three is not able to avoid escape. Here, higher order control algorithms are more efficient in reducing the response of the structure with control forces of the same order of magnitude. It is important to note that the maximum control force required is only about 5% of the static pressure load used in this example and 4% of the static critical load.

As in the case of the harmonic loading, the control system increases the escape load, improving the safety of the structure. For the fifth order control algorithm, for example, we have a escape load 40,12% higher than that without control, as shown in Table 2.

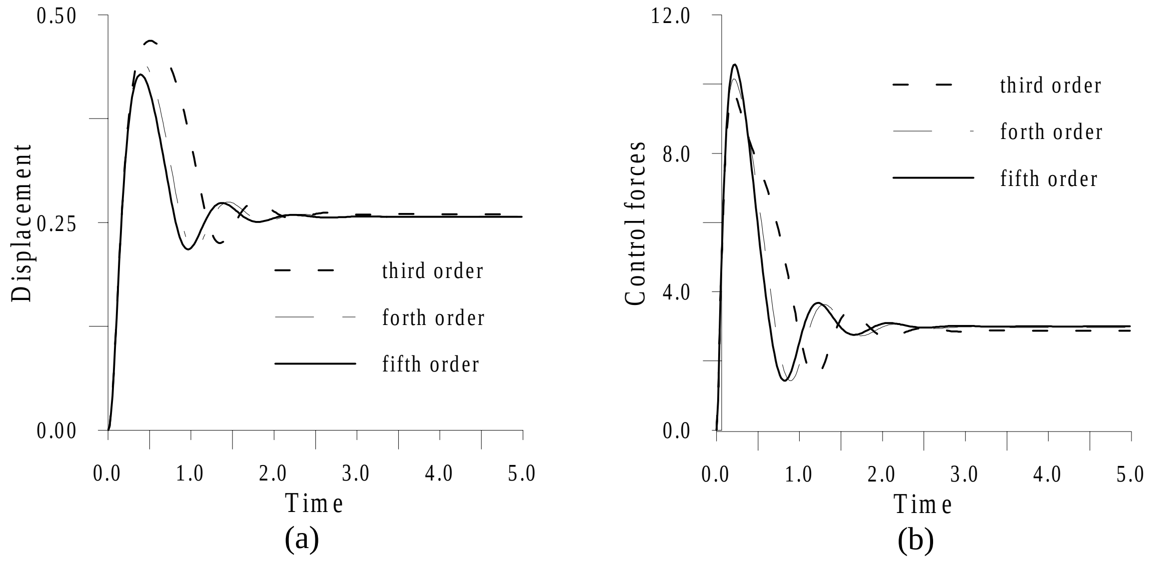


Figure 4. a) Time response and b) control forces for $F_0 = 35$ (step load).

Table 2. Variation of the escape load with the order of the control system (step load).

| Order | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| F0 | 31,95 | 33,67 | 35,29 | 36,41 | 37,65 |
| ΔF0 (%) | 18,91 | 25,31 | 31,34 | 35,50 | 40,12 |

### 3.2 Controlled system with time delay

In this section an input time delay is considered. The system was integrated using a fifth order Runge-Kutta algorithm and the application of the control force was increasingly delayed. The cases of harmonic and impact loading were studied, as well as the free vibration response. For each loading case, both the amount of time delay that made the control system inefficient ($\beta_i$) and the amount that caused escape ($\beta_e$) were computed, using control algorithms of order one to five. The control system is considered inefficient when the rms or the peak value for displacement, velocity or acceleration of the controlled response is greater than that obtained without control,.

For the system under free vibration and considering a starting position with the displacement w = 0,6 and velocity equal to zero, the amounts of time delay that made the control system inefficient ($\beta_i$) were computed. These critical values are shown in Table 3.

Table 3. Critical values of time delay - free vibration.

| Order | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\beta_i$ | 24,00 | 23,60 | 24,74 | 22,69 | 23,02 |

For an harmonic excitation - $F(t) = 10 \sin(12t)$, the obtained critical values of time delay are shown in Table 4 and in Table 5 the results for impact loading, $F(t) = F_0$.

Table 4. Critical values of time delay - harmonic loading.

| Order | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\beta_i$ | 35,07 | 35,43 | 35,22 | 34,91 | 34,71 |
| $\beta_e$ | 40,97 | 41,77 | 45,30 | 41,19 | 40,97 |

Table 5. Critical values of time delay - impact loading.

| Order | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\beta_i$ | 24,00 | 24,00 | 24,84 | 24,84 | 24,71 |
| $\beta_e$ | 42,14 | 36,76 | 33,52 | 29,99 | 29,43 |

One can observe that the system is less sensitive to time delay when subjected to harmonic excitation. For impact loading and free vibration, the critical values are similar. It can also be observed that the critical values of time delay do not change significantly with the order of the control algorithm, except in the case of step loading, when the critical value decreases with the order of the control algorithm.

For the parameters used in this example, no case of instability was found in the simulations using delays from 0 to 100% of the natural period of the structure.

## 4. CONCLUSIONS

The obtained numerical results show that the performance of a control system can be improved using a nonlinear algorithm instead of a linear one, chiefly in the case of strong nonlinearities such as those in the equation for the spherical cap. The control algorithm used here is capable of great reductions of the dynamic response and can also increase the escape load, enlarging the working capacity of the structure without demanding great control forces. Results indicate that the control algorithm adopted can be efficient for non-linear systems, but its efficiency can be reduced by increasing time delay. This emphasizes the importance of considering time delay in the design of active structural control systems.

It is important to note that before applying this strategy for real structures, beside the time delay issue, a number of practical considerations like spillover effects, control-structure iteration, etc... should be studied, since the inadequate application of control forces to a structure could not only render the control ineffective but also cause instability. So, one can conclude that this strategy of nonlinear control is attractive, has a good potential and can be used as a base for the study of more complex structures and for the design of control systems.

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